D.12. Frontier Analysis

In both economics as well as business studies a great deal of research is devoted to the study of efficiency. Firms aim to maximize profits or minimize costs while, in a macroeconomic setting, policymakers aim to increase economic production by utilizing labour and capital more efficiently. The problem is essentially the same: how can output be increased without increasing the inputs used or, reversely, how can inputs be decreased without also decreasing output (Koopmans 1951: 60). Frontier analysis studies these issues by identifying the most efficient firms, industries or countries and comparing the performance of the remainder of the sample to this 'frontier'. This provides insights into how resources are allocated differently, how technology is utilized, and what impact these choices have on final production. This differs from standard productivity analysis, which uses parametric (econometric) techniques to intersect the data with a production function rather than surrounding it with a (production possibility) frontier (Färe et al. 1994: 3). In this chapter, I will show that frontier analysis can be used as a valuable supplement or even replacement for traditional growth accounting.

For students in economic history, frontier analysis is a particularly useful tool for three reasons. Firstly, compared to more traditional growth accounting or productivity analysis it provides a much more detailed picture of the sources of growth (Coelli et al. 2005). As illustrated in the decomposition below, Frontier Analysis does not only capture the effects of technological change but also measures efficiency, which in turn can be decomposed into technical efficiency and allocative efficiency. This breakdown is a welcome addition to, for instance, the measure of TFP, which in historical studies is often quite large and difficult to attribute to specific causes. Secondly, frontier analysis tends to require less stringent assumptions. It can incorporate variable returns to scale, deal with market imperfections, allow for multiple inputs *and* outputs and, most helpfully, the functional form of the production function or the output elasticities need not be known in advance. Thirdly, frontier analysis has so far been underutilized in economic history, a vacuum which begs to be filled. Students looking for a suitable thesis topic can draw inspiration from the myriad of economic history studies that tackle engrossing research questions but rely on outdated methods. The data required for frontier analysis is generally no different to standard productivity studies and thus widely available, be it total economy data on production, labour and capital or a panel of firm-level outputs and inputs.

For educators, frontier analysis is an appealing subject to teach because of its interdisciplinary origins. As it has been applied extensively in both contemporary economics as well as in operations research and management studies, it will appeal to both business and economics students. Most students will also be familiar with the basic math underlying frontier analysis; both micro-economics and operations research cover optimization techniques required to estimate distance functions. In addition, the parametric econometric techniques taught in macroeconomics will make it easier to get started with, for instance, Stochastic Frontier Analysis.

Frontier analysis is not without weaknesses, however, and it has garnered its share of detractors over time, particularly among (macro) economists (Färe et al. 1994: 13).[1] Still, most of these issues can be addressed and, as frontier- analysis methods have advanced, papers in economics and business studies applying them have soared (see e.g. Emrouznejad and Yang 2017). Surprisingly, it has not yet caught on

[1] Reluctance to adopting linear programming techniques, which underlie frontier analysis, mostly concerns the fact that no account is taken of noise or measurement error in the data and that it may be sensitive to outliers.

in the economic history literature. This chapter will help students of economic history contribute by providing a brief introduction to the basic methodology, reviewing some relevant examples of frontier analyses in economic history and listing resources that will help students get started with the productivity analysis.

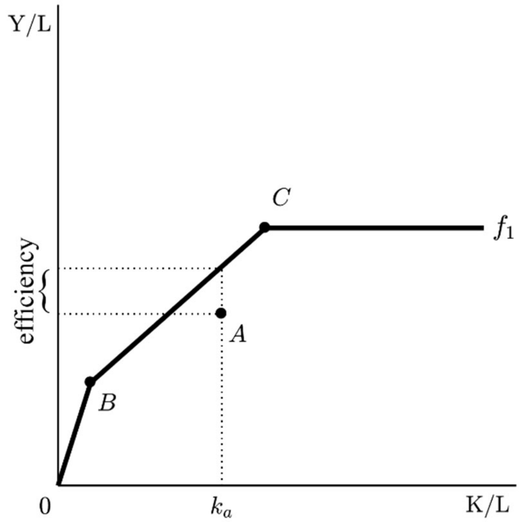
Introduction to Frontier Analysis

The study of efficiency using frontier analysis is ubiquitous, covering "virtually every country and […] every conceivable market or nonmarket production activity." (Färe et al. 1994: 2) The development of frontier analysis in the (macro) economic literature and the micro-oriented business and operations-research literature has been quite distinct, however. To illustrate frontier analysis, this chapter provides a brief introduction of the main non-parametric technique, Data Envelopment Analysis (DEA). This technique is more frequently used in business studies and tends to be the most accessible and the easiest to visualize. Its parametric counterpart, Stochastic Frontier analysis (SFA) is not discussed here, but the 'getting started' section below provides plenty of reference material on this subject.

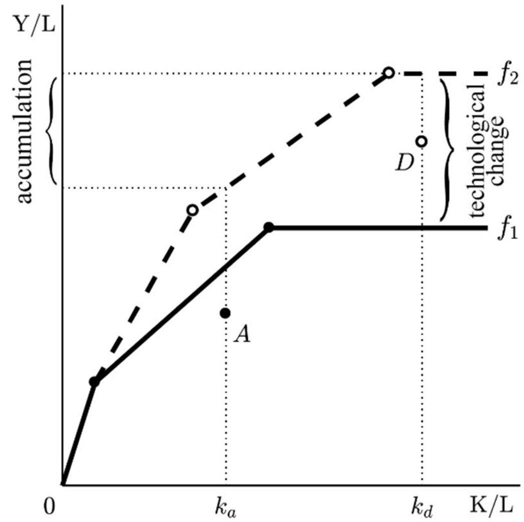
The purpose of a DEA, or any type of frontier analysis for that matter, is to estimate a global production frontier which represents the various 'best practice' production techniques observed for the entire feasible range of input combinations. By tightly enveloping data points with linear segments using mathematical programming methods, the structure of the frontier can be revealed. The global production frontier lends itself more readily to the decomposition of productivity growth as, in contrast to traditional growth-accounting exercises, it distinguishes between both the effects of (global) technological change and relative efficiency change (Färe et al. 1994: 12-13). Efficiency turns out to be a crucial factor in explaining the differences in firms' or countries' growth trajectories as well as in the cross-country divergence of income levels.

Figure 1 depicts a basic example of a DEA involving three producers (A, B, and C) which use two inputs to produce a single output. Assuming constant returns-to-scale, the global production frontier can be presented in $\langle k, y \rangle$ space, where y is labour productivity (Y/L) and k is capital intensity (K/L). The frontier (f_1) for the observations, depicted by the solid line in panel (a), is formed as linear combinations of observed 'best-practice' activities (Salter 1966). As noted in the introduction, an observation is best-practice if increasing any output or decreasing any input is possible only by decreasing some other output or increasing some other input (Koopmans 1951). In this example, only B and C are classified as best-practice techniques. As shown by Färe et al. (1994: 68-9), the identification of these fully efficient observations can be reduced to a basic linear programming problem in the form of a distance function.

Figure 1: Illustration of data envelopment and growth decomposition



(a) frontier



(b) decomposition

More intuitively, producers B and C are both fully efficient. Producer C is more productive (higher Y/L) than B, however, because he applies more capital-intensive production methods (higher K/L). The frontier itself is a subset of all feasible techniques that attain the highest labour productivity for the capital intensity levels they correspond to (Timmer and Los 2005). Panel (a) also shows that the last remaining observation (A) is located below the frontier. Observation A's vertical distance to the frontier indicates the potential for labour-productivity increase through more efficiently using the production factors at hand. This distance can thus be interpreted as a measure of technical efficiency.

The frontier approach can be used in a decomposition of TFP, a process described by Kumar and Russell (2002: 528-529) as "growth accounting with a twist". The example in panel (a) is expanded to include a second period. Panel (b) now includes six observations, shows two frontiers and two inefficient observations (A and D) – which represent the same producer at time 1 and 2 respectively. Labour-productivity change, between A and D, can be decomposed into a change in *efficiency*, *capital accumulation* and *technological change*. The change in *efficiency* is captured by the difference in D's vertical distance to frontier 2 compared to A's distance to frontier 1. *Capital accumulation*, captures the potential change in labour productivity resulting from a shift in the capital-labour ratio. This component represents the average productivity gains or losses as a result of the movement from k_a to k_d along the frontier. Finally, *technological change*, measures the increase in labour productivity as a result of a shift in the frontier. In panel (b) this is captured by the vertical distance between frontier 1 and 2 at D's capital intensity level.[2]

Examples in Macroeconomic History

As with frontier analysis, central to the traditional productivity analysis in economics is how to describe the relationship between inputs and outputs in the aggregate production function. In economic history

[2] For more details on the growth decomposition see Kumar and Russell (2002) and Timmer et al. (2016).

this relationship is still often captured using a standard Cobb-Douglas production function. TFP is generally interpreted as a measure of technology, summarizing how intensively and efficiently inputs are used in production. The typical finding for developed countries is that TFP explains the bulk of growth in output over time. Most famously, Solow (1957) showed that over 80 percent of US labor-productivity growth between 1909 and 1949 came from TFP. Still, even recent studies that employ more encompassing measures of inputs set this share as high as 60 percent (Bakker et al. 2017). In cross-country studies we see that TFP is generally much higher in high-income countries (Feenstra et al. 2015).

This leaves open a lot of questions. How does technology change over time? Why are there such striking differences in technology between countries? How should we interpret and measure technology – is it factor neutral or does it augment one production factor in particular, i.e. is biased towards labour or capital (Bernard and Jones 1996, 1043). The theme of technological change is central in the Economic History literature, ranging from the Great Divergence debate, the analysis of General Purpose Technologies, to rising income inequality. Frontier analysis is able to address most, if not all of these questions.

Using the basic techniques of data envelopment, Allen (2012) shows that technological change was decidedly biased towards higher levels of capital intensity between 1820 and 1990. Most technological progress was achieved by rich countries that employed a great deal of physical capital in their production process, shifting the production frontier only locally. Remarkably, developing countries appeared to be no more productive in 1990 than countries with a similar capital to labour ratio in 1820 (Allen 2012: 9-11). In contrast, traditional productivity analysis generally assumes technological change grows uniformly at all levels of capital intensity. As noted by Timmer and Los (2005: 49-50), the assumption that advances in say (capital intensive) high-speed maglev trains would improve the performance of (capital extensive) rikshaws by an equal factor is clearly wrong, undermining the credibility of growth models based on a classical Cobb-Douglas production function.

Another example of frontier analysis in economic history is the study by Timmer et al. (2016), who use a DEA growth decomposition to show that the gap between German and U.S. labour productivity in manufacturing in 1936 was primarily the result of the inefficient assimilation of modern production techniques in Germany. This finding contradicts traditional explanations that blame the labour productivity gap on the use of different technology and factor endowments. In other words, it was not a lack of capital in German production but the inefficient use of that capital that prevented Germany from catching up to the U.S. prior to the Second World War.

Examples in Microeconomic History

Scholars in operations research, management science and microeconomics also commonly apply frontier analysis. The goals are very similar: identifying the most effective organizations, providing a single summary measure of relative efficiency, handling multiple or constrained inputs and outputs (thus addressing resource scarcity, environmental regulation), or evaluating qualitative factors like customer satisfaction (Färe et al. 1994: 5-6). Crucially, for historians engaged in business or microeconomic research, frontier analysis is particularly well suited to provide insights into the factors that contribute to relative efficiency.

As an example, Lampe and Sharp (2015) evaluate the performance of Danish dairy farmers during the nineteenth century using SFA. They show that efficiency of production was closely linked to education. In addition, they show Danish farmers exploited returns to scale, which allowed them to incorporate new technologies in their production process and increase the use of commercial feedstuffs, which both helped to improve dairy production significantly.

Going back even farther in time, McDonald (2010) assessed productive efficiency of eleventh century English estates using DEA. He shows that efficiency was strongly correlated with the arable/livestock mix and the administrative division of which the estate was part. The estate's tenant-in-chief did not affect efficiency, however, showing that feudal landlords were not conducive to the spread of new technology.

Getting started

There is a wealth of excellent textbooks on frontier analysis. For undergraduate students or graduate students less familiar with microeconomics and linear-programming techniques a good place to start is *An Introduction to Efficiency and Productivity Analysis* by Coelli et al. (2005). More advanced textbooks include *Production Frontiers* by Färe et al. (1994), *Stochastic Frontier Analysis* by Kumbhakar et al. (2000), and the more applied *Data Envelopment Analysis* by Zhu (2016).

Contrary to most econometric methods, frontier analysis is generally not a built-in function in many of the commonly applied software packages (SAS, Stata, R, MatLab, Eviews). Fortunately, there is a host of user-built software tools and packages available on the internet that can perform even the most demanding types of frontier analysis for most of these languages. For those just starting with DEA/SFA or less familiar with computing software, excellent stand-alone programs include the Centre for Efficiency and Productivity Analysis' *DEAP* (Data Envelopment Analysis (Computer) Program) and *FRONTIER*. Neither require a software license and both have outstanding supporting documentation.

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